AR-009-303

DSTO-TR-0183

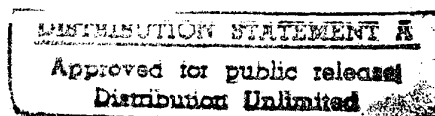# Ship Structural Response Analysis
Spectra and Statistics

B. Phelps
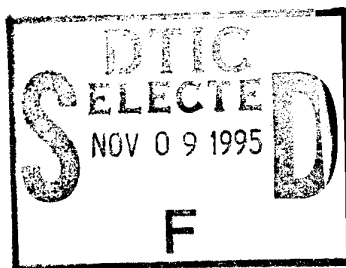
DTIC
SELECTED
NOV 0 9 1995
F

DEPARTMENT ◆ OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

Ship Structural Response Analysis: Spectra and Statistics

B.P. Phelps

Ship Structures and Materials Division
Aeronautical and Maritime Research Laboratory

DSTO-TR-0183

ABSTRACT

Wave loading of ships and maritime structures is a random process and the response of these structures is itself a random process. Much work has been undertaken to better understand the nature of waves and a number of existing techniques are described. Statistical and spectral analysis techniques may be used to quantify the wave loads and the corresponding structural responses. The relationship between the input load and response may then be determined through the use of response amplitude operators and the response of the structure to predicted lifetime extreme loads and fatigue loading then calculated. Standard wave spectra and directional spreading factors are used to enable the seaway to be described mathematically and several standard forms are discussed. The use of statistical methods enables the seaway to be described in terms of a limited number of parameters from which short term and long term probability distributions may be obtained. These distributions may then be used to enable extreme lifetime loads to be estimated.

19951108 077

RELEASE LIMITATION

Approved for public release

DTIC QUALITY INSPECTED 8

D E P A R T M E N T   O F   D E F E N C E

◆

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

Ship Structural Response Analysis: Spectra and Statistics

Executive Summary

Because of the essentially random nature of ocean waves, and hence ship structural responses to the loads imposed by waves, statistical and spectral analysis techniques are commonly used to quantify ocean waves and the corresponding structural responses. Details of these methods and the manner in which they may be applied are summarised in this report.

The report summarises many of the methods used and the way in which they may be applied in practice. For example, the use of wave power density spectrum to quantify ocean waves, the relationship between actual and encounter wave frequencies, the transformation of the spectrum between the two frequency domains and the use of the directional energy spectrum to account for wave spreading are described. For design purposes a number of idealised wave spectrum formulations are available and the applicability of the various forms is discussed. The use of Response Amplitude Operators (RAOs), which provide the ratio of the response spectrum to the input spectrum, are discussed and the need to pay particular attention to the relationship between encounter frequency and actual wave frequency is highlighted.

The long term response of a ship to wave loading over a wide range of operating and environmental conditions determines the fatigue performance of the vessel and the report describes the methods used to make long term fatigue predictions based upon short term observations. The use of short and long term statistical methods is presented and the applicability of particular methods is discussed.

Author

B.P. Phelps
Ship Structures and Materials Division

Bernie Phelps commenced a cadetship with the Department of Defence in 1984 and graduated from University of NSW with B.Eng Naval Architecture (Hons) in 1986 and from University College London with MSc Naval Architecture in 1988. He was employed with the Directorate of Naval Architecture in Canberra in 1987 and 1989 and from late 1989 to April 1992 was the Hull Liaison engineer with the RAN New Submarine Project in Malmö, Sweden. In mid 1992 he started with the then Materials Research Laboratory analysing results of hydrodynamic load trials on RAN Ships and now leads a small team in the analysis and numerical modelling of hydrodynamic loads and ship's structural responses.

Contents

1. Introduction

The in-service loading of a ship may be considered to be broadly made up of two parts, the still water loading and the wave induced loading[1]. Still water loading is a function of the difference between the mass and buoyancy distributions along the length of the ship and so is influenced by the cargo, fuel and stores condition and although essentially random in nature, for most purposes it is normally considered deterministic. Wave loading is a truly random process as it is not possible to predict at any given time the height of a wave relative to the ship.

Traditionally analysis of wave loading has been reduced to a quasi-static condition with the ship balanced on the crest or between two crests of a "standard wave" representing an extreme loading condition, and the load is then superimposed upon the still water load. Although this method has worked well for many years it ignores the random nature of the waves, their dynamic effects and the influence that this has on the total loading and fatigue life of the structure. A better understanding of a ship's response to wave loading is therefore vital in the overall assessment of hull structural performance. Wave loading causes a dynamic ship response which may be considered made up of a whole-ship rigid-body response and a series of deformation responses involving bending and torsion of the hull girder. Structural dynamic theories have therefore been developed whereby the ship is considered to respond as the superposition of responses of increasingly higher order modes of vibration. While such theories are not the subject of this report the idea of the structural response being the superposition of responses at different frequencies is inherent in the methods presented.

Because waves and the subsequent ship's responses are random in nature they are able to be analysed using statistical methods, and much work has been undertaken to better understand the nature of waves through their spectral and statistical distributions and the corresponding responses of the ship. Typically the seaway is described by wave height spectra and linear relationships between the wave height and the ship's response spectra may be developed. These relationships are often called Response Amplitude Operators and their linear nature means that their application is restricted to fairly mild and uniform sea conditions, they do not differentiate between hogging and sagging responses and effects such as breaking waves and slamming are not adequately described. There still remains therefore much work to be done before the relationship between ship and waves is fully understood.

The development of a broadly based expertise in available structural analysis methods is considered essential to maintaining the ability for the RAN to assess the structural performance of existing and future ships. To better understand ships' responses to wave loading trials have been carried out by DSTO on two Royal Australian Navy vessels, HMAS SWAN and SYDNEY so as to monitor and record ships' motions and

---

[1] Other loading types such as grounding, impact and blast loading are not relevant to this discussion and are ignored.

structural responses in a seaway. The data obtained from these trials is used for a number of purposes such as prediction of ship responses to given wave loading conditions, determination of the effects of structural modifications, validation of finite element models and to provide information relating to the fatigue loading of RAN ships. The purpose of this report is therefore to provide an outline of existing analysis techniques employed and the manner in which they may be applied.

2. Wave Analysis

Ocean waves are generated primarily through the action of the wind over the water, and their formation is influenced by such factors as the wind strength and direction, the time for which the wind blows, fetch (the distance over which the wind blows) and topography of the ocean floor. The waves generated form an irregular pattern of wave heights, lengths, directions and frequencies and in order to analyse wave records and corresponding responses a simplified model is adopted. Typically spectral analysis is used to describe the wave energy as a function of wave frequency and the corresponding responses at these frequencies are investigated as outlined in the following sections.

2.1 Encounter Frequency

As a ship moves through a seaway, the frequency at which it encounters the waves is different to the actual wave frequency due to a Doppler shift effect. Figure 1 depicts a ship moving with velocity V_s at an angle ψ to a regular seaway with circular frequency ω. These parameters are related to the *encounter frequency* ω_e of the ship by

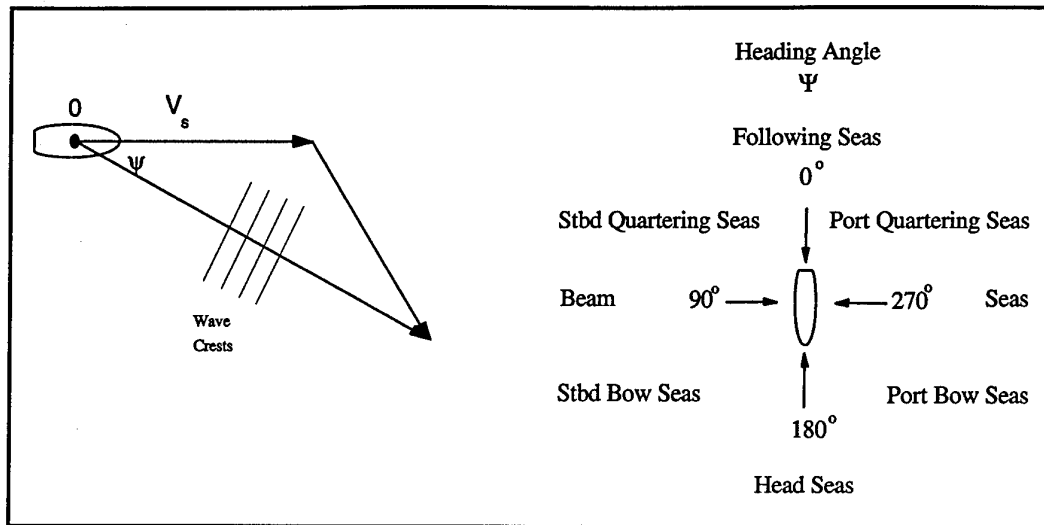$$\omega_e = \omega - (\omega^2 / g)V_s \cos\psi \ . \tag{1}$$

Figure 1. Motion of a Ship Relative to a Regular Wave System

The encounter frequency of the ship is then dependent not only on the wave frequency, but also the speed of the ship and the relative angle between the ship and the seaway and depending on these parameters a number of alternative situations are possible. It can be seen from equation (1) that in bow or head seas ($90°<\psi<270°$) $\cos(\psi)$ is always negative so the encounter frequency is always greater than the wave frequency. If $\psi = 90°$ or $\psi = 270°$ (beam seas) then $\cos(\psi) = 0$ and the encounter frequency is equal to the wave frequency. If, on the other hand, the ship is in quartering or following seas ($0°<\psi<90°$ or $270°<\psi<360°$) then $\cos(\psi)$ is positive and so, depending on the wave frequency, the encounter frequency may be either positive or negative.

In quartering or following seas the encounter frequency will be zero if $\omega = 0$ or $\omega =g/(V_s \cos\psi)$ and will be maximum for $\omega =g/(2V_s \cos\psi)$, in which case $\omega_e = g/(4V_s \cos\psi) = \omega/2$. In deep water the wave velocity (celerity) $c = g/\omega$ so that high frequency waves advance slowly whereas low frequency waves advance rapidly. This means that if the encounter frequency is zero ($\omega \neq 0$) the ship will be travelling at the same speed as the waves, if encounter frequency is positive the waves will be overtaking the ship, and if encounter frequency is negative then the ship is overtaking the waves. The relationship between actual frequency and encounter frequency for quartering and following seas is depicted graphically in Figure 2. It is seen also that if only the absolute value of encounter frequency is known, and $|\omega_e| < \omega_{e\ max}$, as might be the case when encounter frequency is provided by on-board instrumentation, then three wave systems are possible.
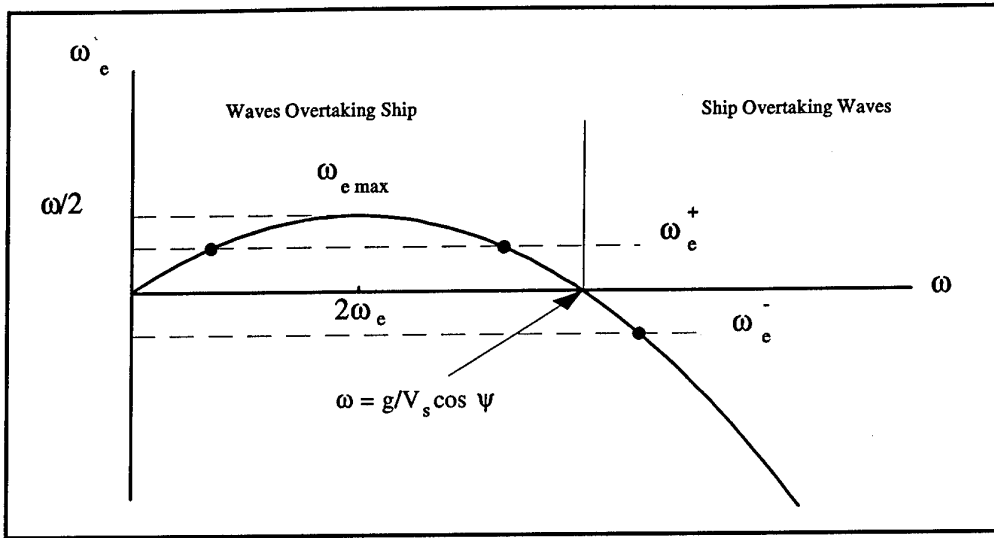
Figure 2. Encounter Frequencies in Following and Quartering Seas.

Whatever the relative direction of the seaway, the possible values of wave frequency ω may be obtained by rearrangement of equation (1), which gives

$$\omega = \frac{g}{2V_s \cos\psi} \left[1 \pm \sqrt{[1 - \frac{4\omega_e}{g} V_s \cos\psi]} \right] \quad \text{rad / sec}, \quad \omega \geq 0 \qquad (2)$$

In cases where more than one solution is possible (i.e. more than one real solution) it will be necessary to consider other information such as on-board observations in order to determine the actual value of ω.

2.2   The Wave Spectrum

Waves are generated through the action of the wind blowing across the surface of the water. Initially ripples are formed and these advance with increasing fetch until waves are created. The wind will continue to act on these waves generating further ripples and waves so that eventually the observed waves at any point are an irregular superposition of waves of different heights, length and frequencies. Under ideal conditions the wave components all travel in the direction of the wind, but this is never actually the case and it is found that the irregular sea has components in many different directions, a phenomenon referred to as wave spreading.

A seaway is an essentially random process and it is assumed that the irregular sea at a particular point may be described mathematically by the linear superposition of a large number of regular sinusoidal waves of different amplitudes and frequency, that is as a

Fourier series. Typically this information is presented as a uni-directional wave spectrum which provides the wave energy density (energy per unit horizontal surface area) as a function of wave frequency. The complete name for such a spectrum is the *Wave Energy Density Spectrum*, however it is usually just referred to simply as the wave spectrum. Sometimes the term point spectrum is used to denote that the spectrum is observed at a fixed point, without consideration of the direction of component waves. To enable components from different directions to be included a directional wave spectrum may be used where the wave energy is assumed to be spread over a range of directional components. The spreading is usually assumed to be symmetric about the primary direction of wave travel (see section 2.4) and so does not accurately describe a bi-directional sea state where there are two main directional components. Also, the superposition of regular waves does not exactly describe an irregular wave system, particularly in higher sea states where non-linear effects such as wave breaking are more pronounced, however it is relatively simple to apply and for many applications is quite accurate.

If an actual ocean wave system is measured as a discrete time record $\{x_r\}$ of wave height over a period of time T at discrete time intervals of Δt, the actual time history may be approximated as the sum of a large number of regular sine waves at discrete frequency intervals of $\delta\omega = 2\pi / T$. A Fourier series is obtained from the discrete time series using the Fast Fourier Transform (FFT) algorithm where the Fourier coefficients identify the magnitude and phase lag of each component wave. Although the ocean wave system is considered statistically stationary at the time of the measurement, the particular time history being examined is unique and will never be repeated. It is more useful therefore to consider the energy spectrum obtained from the Fourier representation as it is ideally identical for adjacent measurements whereas the Fourier coefficients of the component waves will vary from sample to sample due to random phasing. The energy density spectrum is then obtained essentially by forming the complex modulus squared of the Fourier combinations, although additional numerical manipulation known as smoothing is required to provide an accurate estimate of the true spectrum. The steps involved are explained in detail and in summary by Newland (1984) and so will not be repeated here.

The wave spectrum may be expressed in terms of either circular frequency ω (rad/sec) or cyclic frequency f (hertz). In either case the area under the curve must remain unchanged so if $S_\zeta(\omega)$ is the wave spectrum in terms of circular frequency and $W_\zeta(f)$ is the spectrum in terms of cyclic frequency then

$$W_\zeta(f = \frac{\omega}{2\pi}) = 2\pi . S_\zeta(\omega) . \tag{3}$$

A typical measured wave energy density spectrum in terms of cyclic frequency is shown in Figure 3. This spectrum has been obtained through averaging the results of 20 overlapping spectra, each obtained using 2048 data points.
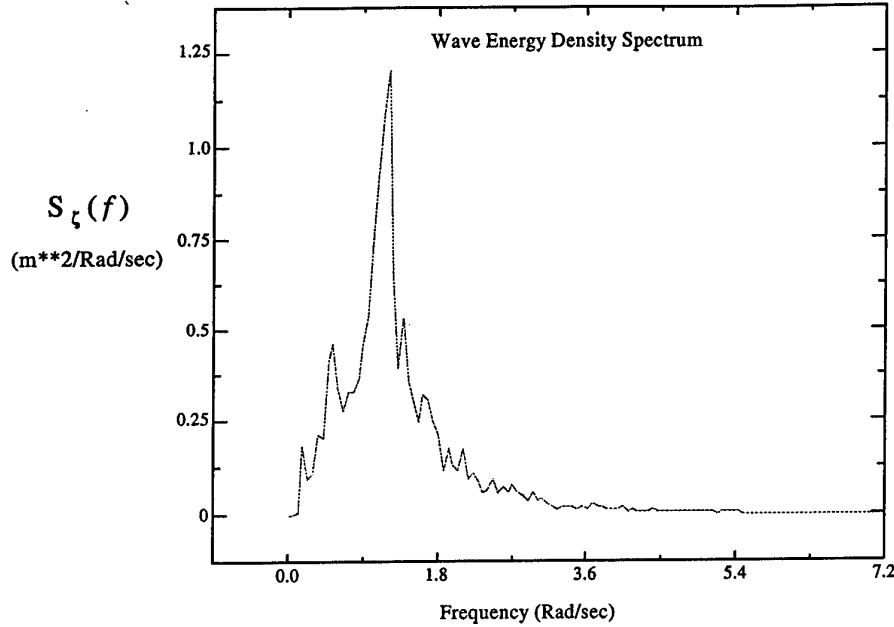
Figure 3. *Typical Measured Wave Energy Density Spectrum*

The area beneath the curve of an infinitesimal strip of width $\delta\omega$ (for cyclic frequency representation) and centred at ω_k is proportional to the energy density of a regular wave of frequency ω_k. This proportionality is true whichever spectral form is used. The units of the spectral density ordinates are $m^2/rad/sec$ and since the energy density of a regular wave of frequency ω_k and amplitude[2] $\zeta_{0,k}$ is $\frac{1}{2}\rho g\zeta_{0,k}^2$, the constant of proportionality must therefore have units of ρg ($kg/m^3 {}^*m/s^2$). The area beneath the wave spectrum for the frequency band centred at ω_k will then be given by[3]

$$\rho gS_\zeta(\omega_k).\delta\omega = \frac{1}{2}\rho g\zeta_{0,k}^2$$

$$S_\zeta(\omega_k).\delta\omega = \frac{1}{2}\zeta_{0,k}^2 \qquad\qquad m^2 / rad / sec. \qquad\qquad (4)$$

The right-hand-side (RHS) of equation (4) is in fact equal to the variance of the wave elevation of the regular sine wave of frequency ω_k and amplitude $\zeta_{0,k}$. That is

---

[2] ζ is used to denote Wave *Elevation* while ζ_0 denotes Wave *Amplitude*

[3] Care should be taken when dealing with published wave spectral data as in some cases the constant of proportionality may be taken to be $\frac{1}{2}\rho g$ in which case $S_{2\zeta}(\omega_k) = 2S_\zeta(\omega_k) = \dfrac{\zeta_{0,k}^2}{\delta\omega}$.

$$E[\zeta_k(t)^2] = S_\zeta(\omega_k).\delta\omega = \tfrac{1}{2}\zeta_{0,k}^2 \; . \tag{5}$$

The total variance of the spectrum is obtained by integration of the left-hand-side of equation (4). Each wave component may be assumed to be independent and therefore provided the number of frequency bands is sufficiently large so that $\delta\omega \to 0$, the variance of the sum approaches the sum of the variances of the individual component waves, and

$$E[\zeta(t)^2] = m_0 = \int_0^\infty S_\zeta(\omega).d\omega \; . \tag{6}$$

That is, the area beneath the spectrum is equal to the variance of the wave system. The notation m_0 is used to refer to the variance as it is in fact the zeroth moment of the spectral area about the origin. Since the variance of the wave elevation is proportional to the energy density of the wave, m_0 provides a good measure of the severity of the wave system, and $\sqrt{m_0}$ is in fact the modal or most probable wave amplitude ζ_m. This is not to be confused with the average wave amplitude $\bar{\zeta}_0$ which is equal to $1.25\sqrt{m_0}$ (see further in section 2.5).

If the spectrum is to be expressed in terms of the cyclic frequency rather than the circular frequency, then

$$W_\zeta(f).\delta f = 2\pi.S_\zeta(\omega_k).\frac{\delta\omega}{2\pi} = \tfrac{1}{2}\zeta_{0,k}^2 \; .$$

and again

$$E[\zeta(t)^2] = \int_0^\infty W_\zeta(f).df \tag{7}$$

2.3 Transformation of the Wave Spectrum

The wave energy spectrum is defined for a fixed point in space and so it is frequently necessary to transform the spectrum to the frame of reference of the ship. The wave energy spectrum must be shifted along the frequency axis so that the frequency interval of $\delta\omega$ at ω_k is transformed into the corresponding encounter frequency interval $\delta\omega_e$. The area under the wave energy spectrum must be the same in each case so that the area of each corresponding incremental strip must be the same. That is

$$S_\zeta(\omega_{e,k}).\delta\omega_e = S_\zeta(\omega_k).\delta\omega$$

and as $\delta\omega$ and $\delta\omega_e$ become infinitesimal

$$S_\zeta(\omega_e) = S_\zeta(\omega).d\omega / d\omega_e \; .$$

So from equation (1) the encounter frequency spectrum is

$$S_\zeta(\omega_e) = S_\zeta(\omega).\frac{g}{g - 2\omega V_s \cos\psi} \qquad \text{m}^2/\text{rad/sec}. \qquad (8)$$

Sometimes it is convenient to consider the wave system in terms of the wave *slope* spectrum, for example when considering angular motions of roll pitch and yaw. In deep water the wave slope amplitude is given by

$$\alpha_0 = \omega^2 \zeta_0 / g \qquad \text{radians.}$$

And in the same way as the wave energy spectrum represents the energy of the regular sine waves that make up the irregular wave, the slope spectrum represents the slopes of the sine waves that make up the irregular wave. Then

$$S_\alpha(\omega_k) = \frac{1}{2}\frac{\alpha_{0,k}^2}{\delta\omega} = \frac{\omega_k^4}{g^2}S_\zeta(\omega_k). \qquad (9)$$

That is, the wave slope spectrum may be obtained by multiplying the wave energy spectral ordinates by ω_k^4 / g^2.

2.4 The Directional Wave Spectrum

So far it has been assumed that the wave energy spectrum has been derived from a record of the surface elevation at a particular point without regard to the direction of the components contributing to the wave system. Invariably the wave system at a particular point will contain contributions from several directions, and the amount of spreading at any particular time and place is dependent upon both the topology of the ocean floor and the past history of the wave system. The *directional wave spectrum* represents both the frequency and the directional spread of energy of the wave system. It is defined such that the wave energy density in the frequency band $\delta\omega$ and the directional band $\delta\phi$ is equal to the energy density of the regular wave of frequency ω_k which is travelling in direction ϕ_i. That is, similar to equation (4),

$$S_\zeta(\omega_k,\phi).\delta\omega.\delta\phi = \frac{1}{2}\zeta_{0,k,i}^2 \qquad \text{m}^2/\text{rad/sec}. \qquad (10)$$

For design purposes it is usual to assume that the spreading is symmetric about the primary wave direction and is distributed over a spreading range of $\pm\phi_{max}$. This is achieved through the use of a cosine spreading factor $C(\phi)$ such that $S_\zeta(\omega_k,\phi) = C(\phi).S_\zeta(\omega_k)$ and where (Lloyd, 1989)

$$C(\phi) = D \cos^{m}(\frac{\pi}{2\phi_{max}}.\phi) \qquad -\phi_{max} < \phi < \phi_{max} \qquad (11)$$

$$= 0 \text{ elsewhere}$$

where m is a positive integer and

$$D = \frac{\pi}{4\phi_{max}}.\frac{1}{\int_0^{\pi/2} \cos^m (\frac{\pi}{2\phi_{max}}.\phi).d\phi}$$

$$= \frac{1.3.5.....m}{2.4.6.\quad(m-1)}\frac{\pi}{4\phi_{max}} \text{ for } m \text{ odd}$$

$$= \frac{2.4.6....m}{1.3.5.\quad(m-1)}\frac{\pi}{4\phi_{max}} \text{ for } m \text{ even.}$$

As the value m increases the energy becomes progressively more concentrated around the primary wave direction, and for design purposes m = 2 and $\phi_{max} = \pi/2$ are usually chosen, as these have been found by Cummins and Bales to be appropriate for typical open ocean conditions (Lloyd, 1989). The International Ship Structures Congress (ISSC) and the International Towing Tank Conference (ITTC) both recommended values of $\phi_{max} = \pi/2$ for maximum spreading angle but with m=4 and m=2 respectively.

The total wave energy distributed over the range $\pm\phi_{max}$ the point spectrum is then obtained by integration over that range.

$$S_\zeta(\omega) = \int_{-\phi_{max}}^{\phi_{max}} S_\zeta(\omega,\phi).\delta\phi \qquad (12)$$

In practice equation (12) is evaluated at discrete intervals of say 15° and then the point spectrum is obtained by numerical integration over the spreading range.

2.5 The Statistics of Irregular Waves

Over a period of 20-30 minutes a wave system at a particular point may be considered statistically stationary and so certain statistical characteristics may be determined. Wave parameters commonly of interest are:

- wave amplitude ζ_0 (metres) and the mean value of many measurements $\bar{\zeta}_0$
- wave height H (metres) and the mean value of many measurements $\bar{H}$
- wave period T_p between adjacent peaks (seconds) and the mean value of many measurements $\bar{T}_p$

- zero crossing period T_z (seconds) and the mean value of many measurements $\overline{T}_z$

Wave height and amplitude are usually defined as being for each event between zero up-crossings and it is also common to refer to a significant wave height $\overline{H}_{1/3}$ which is the mean value of the 1/3rd highest of many wave height measurements. A typical fixed-point wave record and the various wave parameters are shown in Figure 4.
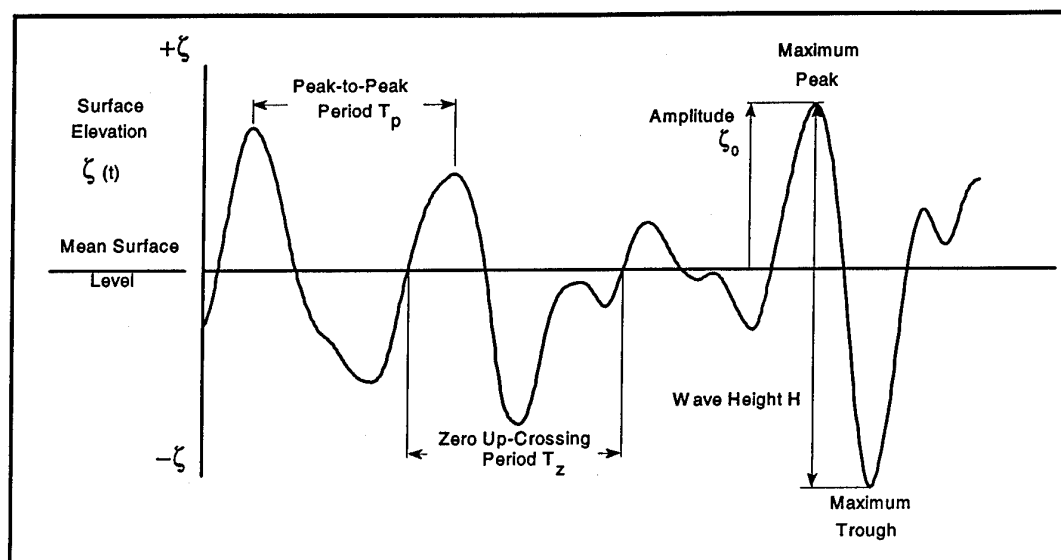
Figure 4. Typical Fixed-Point Wave Record and Definition of Parameters

In equation (6) the variance was obtained by taking moments of the wave spectrum about the origin $\omega = 0$. In a similar manner, higher moments of the wave spectrum may be determined and used to compute a number of wave parameters as follows (Bishop and Price, 1979).

m_0 = variance of the wave elevation
= zeroth moment of the wave spectrum
= $\int_0^\infty S_\zeta(\omega).d\omega$ (m^2)
= the area under the wave energy density spectrum
= mean square wave *elevation* (for zero mean process)

m_2 = second moment of the wave spectrum
= $\int_0^\infty S_\zeta^2(\omega).d\omega = \int_0^\infty \omega^2 S_\zeta(\omega).d\omega$ (m^2/sec^2)
= the area under the wave vertical velocity spectrum

m_4 =   fourth moment of the wave spectrum

     = $\int_0^\infty S_\zeta^4(\omega).d\omega = \int_0^\infty \omega^4 S_\zeta(\omega).d\omega$ (m^2/sec^4)

     =   the area under the wave vertical acceleration spectrum

m_n =   n'th moment of the wave spectrum

     = $\int_0^\infty \omega^n S_\zeta(\omega).d\omega$ (m^2/sec^n)

$\overline{\omega}$ =   average frequency

     = m_1/m_0 (rad/sec)

$\overline{T}$ =   characteristic period (of the component wave of average frequency $\overline{\omega}$)

     = $2\pi\, m_0/m_1$ (sec)

$\overline{T}_p$ =   average wave period between peaks or hollows

     = $2\pi\,\sqrt{(m_2/m_4)}$ (sec)

$\overline{T}_z$ =   average zero-upcrossing period

     = $2\pi\,\sqrt{(m_0/m_2)}$ (sec)

ζ_m =   most probable (modal) wave amplitude

     = $\sqrt{m_0}$ (m)

$\overline{\zeta}_a$ =   average wave amplitude

     = $1.25\sqrt{m_0}$ (m)

$\overline{H}$ =   mean peak to trough wave height

     = $2.51\sqrt{m_0}$ (m)

$\overline{H}_{1/3}$ =   significant wave height (average of 1/3rd highest waves)

     = $4.0\sqrt{m_0}$ (m)

$\overline{H}_{1/10}$ =   average height of 1/10th highest waves

     = $5.05\sqrt{m_0}$ (m)

$\overline{n}$ =   average number of zero upcrossings per unit time

     = $\dfrac{1}{2\pi}\sqrt{(m_0/m_2)}$

The formulae for average periods, heights and amplitudes are strictly only valid if the wave elevation, measured at equal time intervals, is a zero-mean process and has a Normal or Gaussian distribution, and the wave spectrum is narrow banded. This

occurs if the phase angles of the component regular waves have uniform probability between 0 and 2π, and fortunately in the short term (possibly up to several hours in steady conditions), this is approximately true for ocean waves.

The *bandwidth parameter* is defined as

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} = \sqrt{1 - (\overline{T}_p / \overline{T}_z)^2} \tag{13}$$

and is usually small and any errors associated with the above formulas for wave height, period, etc. are also quite small. Ochi (1981) showed for a large number of wave cycles that when determining extreme values with a low probability of exceedance (section 5.3), the error associated with this assumption is quite small even for values of ε up to 0.9.

Empirical relationships have been developed which relate significant wave height and zero-upcrossing period with visual estimates of wave heights and periods. It is generally considered that visually estimated wave heights can be taken as the significant wave height, for a long time it was generally accepted that observed period corresponds to the zero crossing period. Visual estimates of wave period are generally considered to be inaccurate, particularly if the observed sea is bi-directional and made up of a combination of swell and wind waves. The following relationships were derived by Nordenström from a large number of visual observations and are sometimes used to improve the accuracy of visual observations (Lewis, 1989). It should of course always be remembered that there is a considerable scatter in the data used in their derivation, and that visual wave estimation is a subjective assessment.

$$\begin{aligned} \overline{H}_{1/3} &= 1.68 \, H_v^{0.75} \\ \overline{T}_z &= 0.82 \, T_v^{0.96} \end{aligned} \tag{14}$$

(H_v and T_v are the visual estimates of wave height and period respectively.) More recently it has been found (Hogben et al, 1986) that visually observed wave period corresponds more closely to the modal wave period, $\overline{T}_p$ (the period corresponding to the peak of the wave energy spectrum), and that it is related to zero-crossing period by $\overline{T}_p = 1.4 \, \overline{T}_z$.

Joint probability distributions, or scatter diagrams, of wave heights and wave periods for 104 areas covering the globe are provided by Hogben (Hogben et al, 1986). The results of 55 million observations are summarised and are enhanced to account for wind speed and direction. The scatter diagrams provide the joint probabilities of wave heights in increments of one metre and wave periods in increments of one second and are broken down into geographical area, season and 45° directional groupings. Seasonal groupings depend on the local seasonal patterns and directional groupings

define the direction of the wind generated wave component. Wave height and period given in the tables correspond to the significant wave height and zero-crossing period respectively. When processing the raw visual data only those wave height observations which included both wind waves and swell were included and these were combined to give the resultant significant wave height $\overline{H}_{1/3} = (H_w^2 + H_s^2)^{1/2}$. Visual observations of wave period were not used as these are considered to be unreliable. Instead wave period statistics based on the modelling of joint probability of wave period with wave height derived from the analysis of instrument measured data are used.

3. Idealised Wave Spectral Families

Various attempts have been made to establish idealised wave spectral families which would enable designers and hydrographers to define the wave spectrum through a limited number of parameters. Often the use of these spectra is appropriate only to a particular stage of wave generation so care must be taken to use them accordingly. Most of these spectra are uni-modal (singular peak) and take the form

$$S_\zeta(\omega) = \frac{A}{\omega^\alpha} \exp[-B/\omega^\beta] \tag{15}$$

where α and β are empirically derived coefficients while A and B are constants. This may also be written in a non-dimensional form, relative to the modal value of the spectrum $S_\zeta(\omega_m)$, the maximum value given by $dS_\zeta(\omega)/d\omega = 0$ and at which point $\omega_m = (B\beta/\alpha)^{1/\beta}$. Then (Bishop and Price, 1979)

$$\frac{S_\zeta(\omega)}{S_\zeta(\omega_m)} = (\omega/\omega_m)^{-\alpha} \exp\left\{\frac{\alpha}{\beta}[1-(\omega/\omega_m)^{-\beta}]\right\}$$

3.1 Pierson-Mokowitz Spectrum

The Pierson-Mokowitz spectrum is an open water spectrum of the general type of equation (15) but which only depends upon a single parameter V_w, the wind speed. It is intended to describe the point spectrum for fully developed seas and was developed primarily for oceanographic use. It is perhaps of limited use to the naval architect because for fully developed seas to occur there must be large fetch, constant wind speed and direction over several hours and no contamination from swells in other wave generation areas. The spectrum is given by

$$S_\zeta(\omega) = \frac{Ag^2}{\omega^5} \exp[-B(g/V_w)^4] \tag{16}$$

where $S_\zeta(\omega)$ = the spectral ordinate in $cm^2.sec$

ω = frequency in radians/sec

A = $8.10 * 10^{-3}$

B = 0.74

V_w = wind speed in cm/sec measured at 19.5 m above the sea surface

An estimate of the average wave period (UCL, 1988) using this spectrum is given by

$$\overline{T} = 0.81(2\pi V_w / g)$$

and of significant wave height by

$$\overline{H}_{1/3} = 2.12 \times 10^{-2} V_w^2$$

where V_w is in metres/sec.

3.2 The Bretschneider, ISSC and ITTC Spectra

The Bretschneider spectrum is also an open water spectrum of the kind given in equation (15) with $\alpha = 5$ and $\beta = 4$, and is designed to represent both rising and falling seas as well as fully developed seas. While individual point spectra may not match the Bretschneider spectrum particularly well, it was found by Ochi and Bales that the differences tended to disappear through statistical averaging (Lewis, 1989). It is at best to be considered approximate but it does permit both period and wave height to be specified separately. The spectrum is:

$$S_\zeta(\omega) = \frac{A}{\omega^5} \exp[-B / \omega^4] \qquad (17)$$

and the two constants A and B are given by

$$B = 1.25 \, \omega_m^4$$
$$A = 4 \, m_0 \, B$$

This spectrum was adopted by the ISSC (1967) to be used in conjunction with observed wave height and period, which are taken to be the significant wave height and the characteristic period as given in section 2.5, and where

$$A = 173 \, \overline{H}_{1/3}^2 / \overline{T}^4$$
$$B = 691 / \overline{T}^4 \qquad (18)$$

The characteristic period $\bar{T}$ is usually taken as the zero crossing period $\bar{T}_z$. When visually observed wave height and period are used for equation (18) it is recommended that the results given in equation (14) be used to determine significant wave height and zero crossing period before calculating the spectrum according to equations (17) and (18).

The Bretschneider spectrum was also recommended by the 15th International Towing Tank Conference (ITTC, 1978) for average conditions (not fully developed seas) when the significant wave height is the only information available, then in equation (17)

$$A = 8.10 * 10^{-3} g^2$$
$$B = 3.11/\bar{H}_{1/3}^2$$
(19)

3.3 Joint North Sea Wave Project Spectrum

Joint North Sea Wave Project (JONSWAP) was set up to study the growth of waves under conditions of limited fetch and under the effects of shoaling water. A family of spectra was computed from the measurements made by the project and are again of the form given by equation (15) but modified by a frequency dependent (and somewhat complicated) peak enhancement factor. The effect of the peak enhancement factor is to produce a spectrum which is narrower and has higher peaks than the standard Pierson-Mokowitz form, and which is more suitable for storm conditions than fully developed open ocean waves. The spectrum is dependent upon wind speed and fetch, and although it can at times provide a better fit to data than other spectra it does not lend itself to practical use for design or most response analysis purposes. A modified form of the JONSWAP spectrum has been proposed by Det Norske Veritas (UCL, 1988) which enables the spectrum to be calculated using significant wave height and zero crossing period.

$$S_\zeta(\omega) = \frac{H_{1/3}^2 T_z}{8\pi^2} \left(\frac{\omega T_z}{2\pi}\right)^{-5} \exp\left\{-\frac{1}{\pi}\left(\frac{\omega T_z}{2\pi}\right)^{-4}\right\}$$
(20)

3.4 The Ochi 6-Parameter Spectrum

A family of spectra were developed by Ochi and Hubble which generally provides a much better fit to measured wave spectra than do the previously mentioned spectra, particularly in cases where the spectrum is bi-modal (Lewis, 1989). This spectral family has the form

$$S_\zeta(\omega) = \frac{1}{4} \frac{(\frac{4\lambda + 1}{4} \omega_m^4)^\lambda}{\Gamma(\lambda)} \cdot \frac{\overline{H}_{1/3}^2}{\omega^{4\lambda+1}} \exp[-\frac{4\lambda + 1}{4}(\omega_m / \omega)^4] \tag{21}$$

where H_s is the significant wave height, $\Gamma(\lambda)$ is the gamma function and λ is a shape parameter. The spectrum given by equation (21) is uni-modal with the peak becoming sharper with increasing λ. By adding two of these forms together a six parameter spectrum is obtained which was able to better match commonly observed bi-modal spectra. Then

$$S_\zeta(\omega) = \frac{1}{4} \sum_j \frac{(\frac{4\lambda_j + 1}{4} \omega_{mj}^4)^{\lambda_j}}{\Gamma(\lambda)} \cdot \frac{\overline{H}_{1/3,j}^2}{\omega^{4\lambda_j+1}} \exp[-\frac{4\lambda_j + 1}{4}(\omega_{mj} / \omega)^4] \tag{22}$$

where j=1,2 stands for the lower and higher frequency components, respectively. The six parameters $\overline{H}_{1/3,1}, \overline{H}_{1/3,2}, \omega_{m1}, \omega_{m2}, \lambda_1$ and λ_2 may be determined numerically by minimising the difference between the measured spectrum and the calculated spectrum. Although bi-modal spectra are able to be represented, no information is given as to the relative directions of the waves. For design purposes it is considered more appropriate to use the ISSC spectrum, as although it may not be as accurate for specific spectra, it adequately describes the statistical average and is simpler to apply.

4. Response Amplitude Operators

The relationship between the wave spectrum and the ship response (typically strain or motions) spectrum, $S_\Re(\omega_e)$, is often assumed linear and is expressed in terms of Response Amplitude Operators. The RAO is a frequency dependent function which provides the ratio of the response spectrum to the input spectrum over the entire frequency range. It is the modulus squared of the frequency response function or transfer function, and does not therefore contain any phase information between the response and the input. As the ship's response is always in terms of encounter frequency the relationship is written as

$$S_\Re(\omega_e) = |H_{\Re\zeta}(\omega_e)|^2 \cdot S_\zeta(\omega_e) = RAO.S_\zeta(\omega_e) \tag{23}$$

However as has been shown in Section 2.1 there is not always a one-to-one relationship between the encounter and actual frequency. For the range of headings between $90° < \psi < 270°$ there is a one-to-one relationship in the transformation from ω to ω_e, but for quartering or following seas this is not necessarily the case. It can be seen from

equation (8) that as ω approaches $g/2V_s\cos\psi$ the denominator approaches zero and hence a singularity occurs. The evaluation of the spectra and RAO must be broken down into three stages as depicted in Figure 2 and a full description of the process is provided by Lewis.

RAOs may be calculated either theoretically using strip theory programs, or may be derived experimentally by monitoring of ships or ship models. The concept of wave spreading was raised in section 2.4 where the use of a spreading factor was applied to the wave spectrum to allow for the different directions of wave components. When evaluating the response spectrum the directional wave spectrum may be used to allow for wave spreading, so then

$$S_{\Re}(\omega_e) = \int_{-\phi_{max}}^{\phi_{max}} |H_{\Re\zeta}(\omega_e)|^2 . S_\zeta(\omega,\phi).\delta\phi \ . \tag{24}$$

In practice equation (24) is evaluated at discrete intervals, of say 15° and the total response spectrum is obtained by numerical integration over the spreading range.

As mentioned in section 2.2 the use of a linear wave spectrum does not exactly describe an irregular wave system, particularly in higher sea states. Similarly the response of a ship to the seaway is not exactly linear, especially to the higher sea states, and so the response spectrum provided in equation (24) will not be exactly correct. When strip theory programs are used during the design process, the calculation of the RAOs involves even more approximations as further simplifying assumptions are used in the calculations of the wave spectrum and the RAOs. Furthermore strip theory calculations do not usually differentiate between hogging or sagging responses which are in practice quite different. The end result is that the calculated response spectrum typically over estimates the response of the ship for higher wave loadings, particularly the hogging response (Clarke 1986). One approach to try to improve these predictions is to carry out either model or full scale tests on ships to monitor their response to the seaway and thereby determine experimental RAOs. These RAOs may then be used instead of those from strip theory calculations to determine the ship's response to a particular seaway, or alternately to modify the calculated RAOs so that the strip theory program may more accurately determine the ship's response. This modification would typically take the form of a scaling factor, and it would be possible to use this scaling factor in the assessment of responses of geometrically similar ships.

5. Probability Formulae

So far, the discussion has revolved around only short term assessment of sea state conditions and the corresponding ship's responses. As the sea state as well as the ship's speed, position, orientation and other loading conditions, are continuously changing, it is impossible to know in advance with any certainty all conditions that the

ship will encounter, and even if it were, it would be totally impractical to assess the ship's response to all of these conditions. To be able to design safe structures the designer is therefore required to make predictions regarding the lifetime loading conditions such as the fatigue cycling and extreme loads on the ship. As wave loading is of a random nature these predictions are based upon wave and response statistics and so an understanding of the relevant probability theory and it's application is required.

5.1 Short Term Probabilities

As already mentioned in section 2.5 the short term distribution of wave elevation can be approximately described by the Gaussian distribution provided the spectrum bandwidth is narrow. Likewise the linear response of the ship, such as strain or ship motions, can be assumed to follow a Gaussian distribution. Figure 5 shows a typical histogram of strain response on the main deck of a frigate measured at equal time intervals over a period of several minutes. The corresponding Gaussian distribution curve is also plotted and it is seen that while the curve provides a reasonable approximation to the data, it is not a true Gaussian process. As is quite often the case the level of sagging responses is higher than for the hogging responses due to non-linear effects such as flared bow and stern sections. The measured response may also be affected by the location of the strain gauge as shear lag effects may reduce the apparent compression loading.
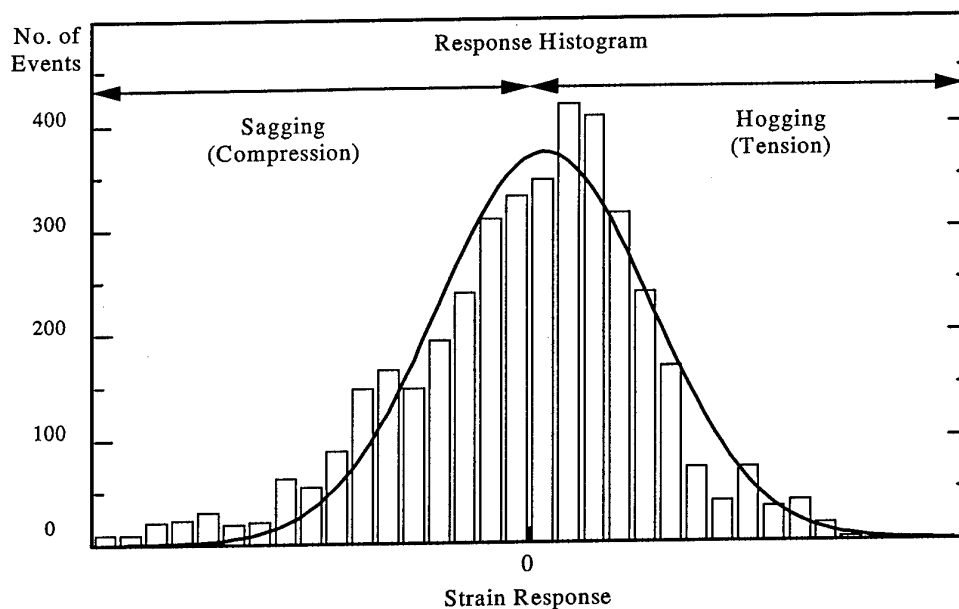

Figure 5. Strain Response Histogram and Corresponding Normal Distribution

The Gaussian probability density function of a continuous random process is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \tag{25}$$

where μ is the mean and σ is the standard deviation of the process. In relation to wave elevation and corresponding response analysis the process will have a zero mean value[4] so then the probability that an individual measurement $\xi = x/\sigma$ will fall between the values ξ_1 and ξ_2 is given by

$$F(\xi_1 \leq \xi \leq \xi_2) = \frac{1}{\sqrt{2\pi}} \int_{\xi_1}^{\xi_2} e^{-\xi^2/2} \, d\xi.$$

Intergration of the RHS of the equation is difficult and so values are obtained from tables.

Usually it is the peak value of the process rather than the variation with time that is of interest, and whereas the short term distribution is approximated by the Gaussian distribution, it is found that the short term distribution of the peak-to-trough (double) amplitudes is approximated by a Rayleigh distribution.

$$f(\xi_a) = \xi_a e^{-\xi_a^2/2}. \tag{26}$$

Note that in relation to a ship's responses to ocean waves, there will actually be a different distribution for the positive and the negative amplitudes if they are considered individually. In other words the hogging and sagging responses are different, and this is well documented (e.g. Clarke, 1986). However for the present it is assumed that the Rayleigh distribution is of the peak-to-trough amplitude, or double amplitude, so that the mean value of the process is zero. If the single amplitude distribution is to be obtained it must be remembered that the mean value is no longer zero. The probability that the double amplitude will fall between ξ_1 and ξ_2 is then

$$F(\xi_1 \leq \xi_a \leq \xi_2) = \xi_a \int_{\xi_1}^{\xi_2} e^{-\xi^2/2} \, d\xi_a$$

$$= e^{-\xi_1^2/2} - e^{-\xi_2^2/2}$$

---

[4] It is assumed that in the analysis of response data, such as recorded strain gauge data, the analysis begins by adjusting the data values so that a zero mean process is achieved.

while the probability that the double amplitude will exceed a certain value ξ_1 gives *the short term probability of exceedance in one cycle*

$$P_{S1} = F(\xi_a > \xi_1) = e^{-\xi_1^2/2}. \tag{27}$$

5.2 Long Term Probabilities

What is usually of interest is the ship's long term response to loading over a wide range of operating conditions and environments, so as to be able to predict the fatigue cycling and expected maximum lifetime loading on the vessel. The long term cumulative probability distribution[5] of loading is therefore required. To achieve this it is first necessary to obtain and analyse a large database of information relating to various sea states likely to be encountered, as well as information relating to the ship's responses to these sea states. For this purpose monitoring trials are often carried out on ships, (e.g. see appendix 2 of Lewis and Zubaly, 1981) and then the data must be analysed so as to determine the long term probability distribution. Typically the following steps will be taken:

(i) Full scale ship's trials are conducted over a broad range of sea state and ship's operating conditions to obtain a comprehensive set of short term response data.

(ii) Each short term record is summarised using histograms of response level versus frequency, and the spectral properties and Rayleigh distribution for each may be calculated.

(iii) The cumulative probability distributions are then determined based upon either the modal (most probable) or extreme wave height values from these records. Commonly a theoretical distribution, such as the Weibull distribution, is fitted to the calculated distribution for the purpose of extrapolation to longer term responses.

5.2.1 Histogram Analysis

Histograms enable the large amount of data collected during the short term trials to be summarised by recording the measured responses in certain pre-determined ranges against their frequency of occurrence. By measuring response cycles over a long period of time and a range of operating conditions, a history of responses and their relative frequency of occurrence is obtained. The histograms provide the probability density of occurrence of particular response levels while the cumulative distribution

---

[5] It is to be remembered that the probability density function may be obtained by differentiation of the cumulative probability distribution.

can be obtained by numerical integration of the histogram data. If all of the histogram data from all of the trials is combined the cumulative distribution gives a plot of response level versus probability of occurrence over the life of the trials. When plotted on a semi-logarithmic plot the data will often approximately follow a straight line, except possibly at the lowest response levels. The distribution obtained through this approach is limited to the time over which the data is collected and should not be used directly for extrapolation to longer periods as the results are limited to the actual weather and sea conditions encountered, the ship's structural design and characteristic response to the seaway. The results obtained are therefore biased towards the actual conditions of the trials and do not necessarily give correct weighting to all sea and operating conditions which may be encountered during the life of the ship. Also, comparative trials have shown that responses vary between similar ships of the same class and even between port and starboard locations on the same ship (e.g. Hoffman and Lewis, 1969).

The histograms are therefore useful in the analysis of existing ships with long histories of voyage monitoring but should not be applied directly to other ships or in ship design unless variations in probable sea and operating conditions can be properly accounted for. It may also be necessary to make certain assumptions as to the form of the long term distribution so as to be able to make consistent predictions of the long term responses and two common methods are used; extrapolation based upon either modal or extreme wave height values (Liu et al, 1981).

5.2.2 Extrapolation Based upon Modal Values

The extrapolation from measured responses to longer periods may be based upon the assumption that the short term distribution of peak values within each record follow the Rayleigh distribution and that the modal or most probable values of the records themselves follow some form of probability distribution. For a zero-mean process the modal *amplitude* is equal to the standard deviation σ and so completely defines the Rayleigh distribution. More typically the modal or most probable wave *height* value is used when describing the sea conditions and so the long term probability density function of the modal wave height values may be given by $\tilde{g}(\sqrt{E_H})$ where $\sqrt{E_H} = \sqrt{2\sigma^2}$. The long term probability density function is defined to account for the variations due to different sea states, the ship's speed, heading, loading condition and so on[6].

The joint long term probability $\tilde{p}(x, \sqrt{E_H})$ is the joint probability of a particular value x (typically bending moment or stress level) with $\sqrt{E_H}$, and is by definition

---

[6] The overscore ~ is used to indicate that $\tilde{g}(\sqrt{E_H})$ is the probability distribution obtained from the recorded data.

$$\widetilde{p}(x, \sqrt{E_H}) = \widetilde{f}(x | \sqrt{E_H}) \cdot \widetilde{g}(\sqrt{E_H}) \qquad (28)$$

where the probability density function $\widetilde{f}(x | \sqrt{E_H})$ is the conditional probability of x for given $\sqrt{E_H}$ and is assumed to follow the Rayleigh distribution.

Different theoretical forms have been suggested for $\widetilde{g}(\sqrt{E_H})$ such as the Gaussian, Weibull and Gumbel distributions. There appears little theoretical justification for any of these forms, other than that they work reasonably well, although the Gaussian distribution tends to over estimate the larger, less frequent values, while the Weibull distribution gives a greater weighting to the lower values. The Weibull distribution appears to be gaining broad approval as it is found that much of the fatigue damage on ships is caused by these more moderate loads which occur with greatest regularity. However it should be noted that it is not essential that $\widetilde{g}(\sqrt{E_H})$ be fitted to any particular theoretical distribution.

The long term probability that x will exceed the level x_0 is then given by substituting the Rayleigh function into equation (28) and integrating with respect to x, which gives

$$P_{L1}(x \geq x_0) = \int_0^\infty e^{-(x_0 / \sqrt{E})^2} \cdot \widetilde{g}(\sqrt{E_H}) \cdot d\sqrt{E_H} . \qquad (29)$$

If $\widetilde{g}(\sqrt{E_H})$ is assumed to follow one of the above standard distributions, certain parameters which define the distribution must be evaluated and this is done by fitting the recorded data to the selected distribution using a suitable error minimisation technique. Numerical evaluation of equation (29) is then usually required, for which an upper limit of five to six times the standard deviation has been shown to provide the minimum value to ensure sufficient accuracy to the final result (Hoffman and Lewis, 1969).

Evaluation of the integral (29) provides the long term probability that *on average* the level x_0 will be exceeded just once in the lifetime of the ship as illustrated by point A in Figure 6. But if many similar ships are considered it would be found that in some cases the level x_0 will not be exceed while in other cases x_0 will be exceed more than once. In fact there is a 67% (Lewis and Zubaly, 1981) probability that this will be the case, which is unacceptably high for design purposes. A probability that the given level x_0 is not exceeded except with an acceptably low level of risk (such as 0.01) by any one of a large number of ships of the same class is therefore required. This *design lifetime probability*, P_{LN} is given by the product of the cumulative probability at the design level x_D times the number of cycles and illustrated by point B in figure 6. The level x_0 will then not be exceeded over the lifetime of the vessel with a probability of $P_{LN} = N * P_{L1}(x_D)$. For example if a value such as $N = 3.0 \times 10^{-7}$ is chosen with a lifetime design probability of 0.01, then the design response value x_D is obtained reading off the curve at $P_{L1}(x) = P_{LN} / N$ i.e. at $P_{L1} = 3.0 \times 10^{-9}$
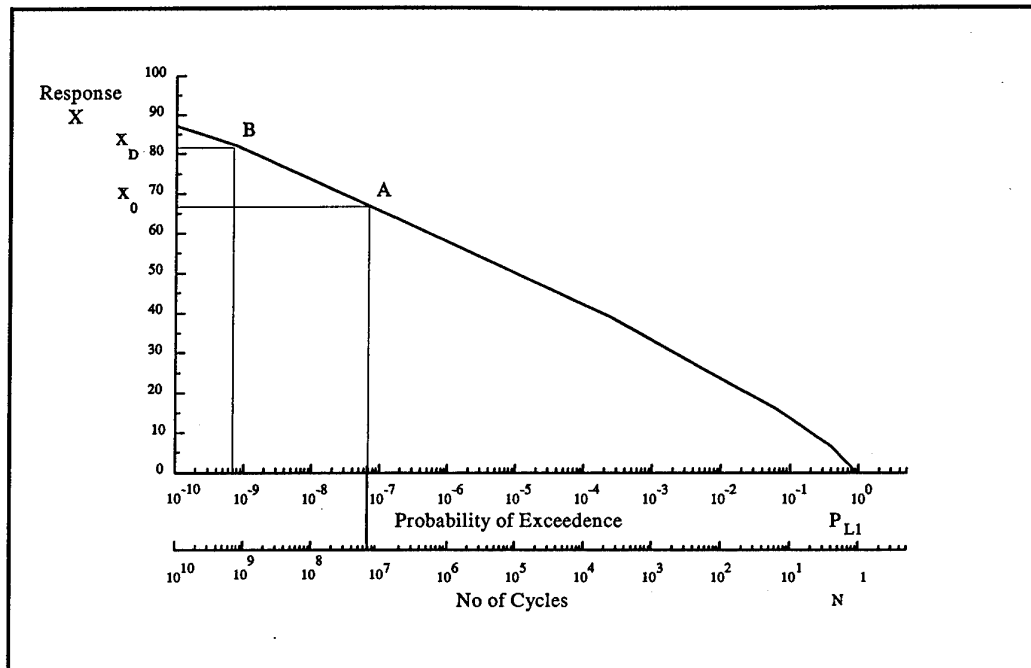
Figure 6. Long Term Probability of Exceedance Based Upon Modal Values

5.2.3 Extrapolation Based upon Extreme Values

Typical of common engineering practice, extrapolation to determine lifetime loading may also be based upon extreme, rather than modal, values. This approach requires potentially less data collection effort as it is only the extreme value from each data sample which is of interest. However the method may also be less reliable as not all the sample information is used, although that this is so is not clearly the case.

If the number of data samples from short term trials is n then a probability distribution of the extreme values may be determined. For each sample there is a probability $F_i(X_i \leq x_0)$ that the extreme response X_i is less than a particular value x_0 in any one cycle. So for the n samples the probability that each of these extreme response in any one cycle is less than x_0 is the same as the probability that the maximum value $\eta = \max(X_1, X_2, \ldots X_n)$ is less than x_0 and is given by

$$G(\eta \leq x_0) = F_1(X_1 \leq x_0) \cap F_2(X_2 \leq x_0) \cap \ldots \cap F_n(X_n \leq x_0) \qquad (30)$$

That is, the cumulative probability distribution $G(x)$ is given by

$$G(x \leq x_0) = \tilde{F}_i^n(x_0) \tag{31}$$

where $\tilde{F}_i(x_0)$ is a shorthand way to denote the combined probability distribution functions of the individual variables, each of which are evaluated at the chosen peak value x_0. Similarly the probability that each extreme response is greater than x_0 will be

$$G(x > x_0) = [1 - \tilde{F}_i(x_0)]^n . \tag{32}$$

The result given in (32) is termed the *long term probability of exceedance in one cycle* and is denoted P_{L1}. What is needed, however, is the probability that x is greater than x_0 at least once in a lifetime of N cycles. So, assuming n is large enough to enable the long term distribution G(x) to be given by $\tilde{F}_i^n(x_0)$, by defining the probability per cycle $G(x > x_0) = P_{L1}$ as "success" and $G(x \leq x_0) = 1 - P_{L1}$ as "failure" then by application of the Binomial distribution it can be shown that for large N that the *long term probability of exceedance in N cycles* is

$$P_{LN}(x > x_0) \cong 1 - e^{-NP_{L1}} . \tag{33}$$

It can be seen from equation (33) that if $P_{L1} = 1/N$ then $P_{LN} = 1 - e^{-1} = 0.632$, so that in a similar fashion to section 5.2.2. there is a 63% chance that the value x_0 will be exceeded, which as before is unsatisfactory. The cumulative probability density function $g(x) = G'(x)$ is related to the initial distribution function F(x), and the modal or most probable value x_m of g(x) is in fact the distribution of P_{LN} for the case where $P_{L1} = 1/N$ (Ochi, 1981), as shown in figure 7.

If the initial probability density function f(x) is assumed to be Rayleigh distributed, then it is shown that

$$x_m = \sqrt{2\sigma^2 \ln(N)} \tag{34}$$

where σ^2 is the variance associated with the Rayleigh distribution f(x). Ochi (1981) suggests the use of a risk parameter $\gamma = P_{LN}$ to be applied to the number of lifetime loading cycles N, so that a more conservative estimate of the extreme value for design purposes x_D is given by

$$x_D = \sqrt{2\sigma^2 \ln(N/\gamma)} \tag{35}$$

which combining (34) and (35) gives

$$x_D = \frac{x_m \sqrt{\ln(N/\gamma)}}{\sqrt{\ln(N)}}.$$ (36)

If for example $\gamma = 0.01$, then equation provides (36) the design extreme value which is not expected to be exceeded with 99% certainty. It can be seen from equation (35) that the design extreme value appears to be more sensitive to the correct evaluation of σ^2 than it is to either N or γ.
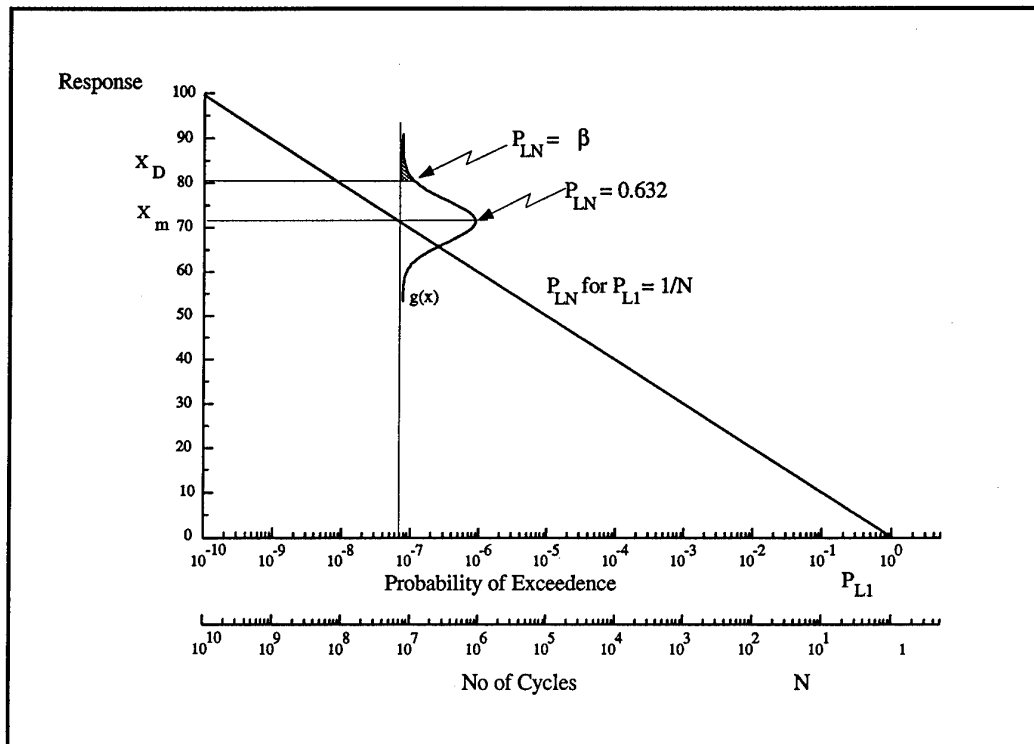
Figure 7. Long Term Probability of Exceedance Based Upon Extreme values

5.2.4 Determination of the Long Term Distribution G(x)

In the above analysis where extrapolation is based either upon the modal value $\sqrt{E}_H$ or the extreme values, it is necessary to determine the cumulative long term probability distribution G(x) in order to be able to determine the design load for the ship

The wave loading of a ship is dependent not only on the sea conditions, but also the ship's speed, relative heading and loading condition, and throughout the life of the ship there will be a certain probability of occurrence associated with any particular

value, or range of values, of each of these parameters. The short term probability calculated from trials results in accordance with equation (27) relates only to one particular set of conditions, which will occur with a particular probability during the life of the ship. Therefore in order to use the trials information to determine the long term distribution it must be weighted according to these probabilities. Obviously a large amount of data and calculations are required to calculate the desired probability and to simplify the problem each parameter is generally considered to be independent, although this is not strictly correct. The long term cumulative probability may then be calculated by the summation of the short term probabilities weighted by the relative frequency of sea state and the relative frequency of the ship's operating state. That is the long term probability distribution G(x) is given by

$$G(x > x_0) = \int_0^\infty \int_0^{2\pi} \int_0^\infty \int_0^\infty \int_0^\infty P_{S1}(x > x_0 : V_s, \psi, H_{1/3}, T_z) . P_{SS}(H_{1/3}, T_z) . P_O(V_s, \psi, \Delta) . dV_s . dH_{1/3} . dT_z . d\psi . d\Delta$$

(37)

where

- P_{S1} is the short term probability of exceedance and is dependent upon ship's speed, relative heading and sea state, characterised by significant wave height and zero crossing period.

- P_{SS} is the probability of encountering a particular sea state characterised by significant wave height and zero crossing period and which is determined by voyage analysis and reference to wave atlases.

- P_O is the probability of the ship's operating condition characterised by ship's speed and relative wave heading (which are themselves a function of sea state although this is often ignored) and displacement. (Note: The effect of ship's speed on bending moment is often quite small (Lewis and Zubaly, 1981) and is often ignored by assuming the highest possible speed for a given set of conditions.)

Although equation (37) is shown as a continuous integral, in practice only a limited number of different combinations will be considered as the level of calculation becomes excessive. The probability of sea state conditions may be determined from wave atlases, which use observations of wave height and period (such as Hogben et al, 1986) to predict the number of waves per 1000 within particular wave height and period bands (see section 2.5).

The probability of the ship's operating condition must consider the distribution of ship's displacement, ship's speed and relative direction of the waves compared to the ship's heading. This is determined by voyage analysis which considers the time that a ship will spend in a particular area, it's speed, heading, displacement and the relative seaway direction. For specific purpose ships such as bulk carriers the response to a particular seaway will vary with loading condition, and so for such ships and others

designed for a particular route there will be a bias towards particular speeds and displacements for particular legs of the journey, and also probably towards particular relative seaway directions. For more general purpose and Navy ships it is generally assumed that each relative wave direction will occur with equal probability so this element of the calculations may be omitted, although the probability of speeds and displacements needs to be considered.

6. Closing Remarks

The study of waves and wave induced loading is a complex task as ocean waves are a random phenomenon where the waves form an irregular pattern of wave heights, lengths, directions and frequencies. Simplifying techniques such as spectral and statistical analyses are used so as to be able to quantify wave properties and the ship's responses to them. Some of the relevant theories have been presented in this paper along with the standardised methods with which they may be applied in the analysis of ships' structural responses to wave loading.

The relationship between actual and encounter wave frequencies is discussed and the effect of relative wave direction upon this relationship is highlighted. The discussion also includes the development and use of wave spectra, the different forms that these spectra may take, the transformation of spectra between frequency domains and the use of directional spectra to account for wave spreading. The typical parameters which may be used to quantify waves and their relationship to the wave spectra are tabulated. Idealised wave spectra used for design and analysis purposes are presented and their applicability is discussed.

The ship's response to waves is often assumed to be linear and is expressed in terms of Response Amplitude Operators. The RAO is a frequency dependent function which provides the ratio of the response to the input spectrum over the frequency range of interest. The development of RAOs requires particular attention to the relationship between encounter frequency and actual wave frequency as there is not always a one-to-one relationship between the two.

The long term response of a ship to wave loading over a wide range of operating conditions and environments will ultimately determine the fatigue performance of the vessel. As the sea state and ship's operating condition are continuously changing it is necessary to make long term fatigue predictions based upon short term observations. The general statistical approach to short term and long term use of such data is presented and the applicability of particular methods is discussed.

7. References

Bishop R.E.D. and Price W.G.
"Hydroelasticity of Ships", Cambridge University Press, 1979

Brown J.C.
"Extreme Wave Induced Stresses in Warships and Their Correlation with Sea State", AMTE(S) TM81407, AMTE(Dunfermline) June 1981.

Clarke J.D.
"Wave Loading of Warships", Journal of Naval Science, Vol 12, No.4 1986.

Hoffman D. and Lewis E.V.
"Analysis and Interpretation of Full-Scale Data of Midship Bending Stresses of Dry Cargo Ships", Ship Structures Committee Report SSC-196, Dated June 1969.

Hogben N., Dacunha N.M. and Oliver G.F.
"Global Wave Statistics", British Maritime Technology, 1986

Lewis E.V., Editor
"Principles of Naval Architecture", Second Revision, The Society of Naval Architects and Marine Engineers, Jersey City, 1989.

Lewis E.V. and Zubaly R.B.
"Predicting Hull Bending Moments for Design", SNAME Extreme Loads Response Symposium, Arlington, VA, Oct 19-20, 1981

Liu D., Chen H. and Lee F.
"Application of Loading Predictions to Ship Structure Design: A Comparative Analysis of Methods", SNAME Extreme Loads Response Symposium, 19-20 Oct 1981.

Lloyd A.R.M.J.
"Seakeeping, Ship Behaviour in Rough Weather", Ellis Horwood Ltd, 1989.

Mansour A.E.
"Probabilistic Design Concepts in Ship Structural Safety and Reliability", Trans. S.N.A.M.E. 1977.

Newland D.E.
"An Introduction to Random Vibrations and Spectral Analysis", 2nd Ed, Longman Scientific and Technical, UK, 1984.

Ochi M.K.
"Principles of Extreme Value Statistics and their Application", SNAME Extreme Loads
Response Symposium, Arlington, VA, Oct 19-20, 1981.

UCL M.Sc. (Naval Architecture)
University College London M.Sc. Hydrodynamics Course Notes, 1988

Nomenclature

A	constant used with standard wave spectra
B	constant used with standard wave spectra
c	wave celerity
$C(\phi)$	cosine spreading factor
D	variable used in determination of spreading factor C
e	natural logarithm, 2.71828
E[ ]	variance
$\sqrt{E}_H$	modal wave height
f	cyclic frequency
$f(x), g(x)$	probability density function
$F(x), G(x)$	cumulative probability distribution
g	acceleration due to gravity
$\tilde{g}(\sqrt{E}_H)$	long term probability density function of modal wave height based upon measured values
H	wave height
$\overline{H}$	mean wave height
$\overline{H}_{1/3}$	significant wave height
$\overline{H}_{1/n}$	average height of 1/n'th highest waves
H_s	visual estimate of swell wave height
H_v	visual estimate of wave height
H_w	visual estimate of wind generated wave height
$H_{\Re\zeta}(\omega_e)$	transfer function of response with wave energy spectrum
m	positive integer
m_0	zeroth moment of the spectral area about the origin
m_n	n'th moment of the spectral area about the origin
N	number of cycles
n	positive integer

$\bar{n}$	average number of zero upcrossings per unit time
P_{L1}	average lifetime probability of exceedance in one cycle (modal value formulation), long term probability of exceedance in one cycle (extreme value formulation)
P_{LN}	design lifetime probability of exceedance (modal value formulation), long term probability of exceedance in N cycles (extreme value formulation)
P_O	probability of the ship's operating condition
P_{S1}	short term probability of exceedance in one cycle
P_{SS}	probability of encountering a particular sea state
$\tilde{p}(x, \sqrt{E}_H)$	joint long term probability density function of x with $\sqrt{E}_H$
RAO	Response Amplitude Operator
$S(\omega)$	energy spectrum in terms of circular frequency (subscript denotes measured parameter)
$S_\zeta(\omega, \phi)$	directional wave energy spectrum in terms of circular frequency
T	period of time
$\bar{T}$	"characteristic period", see section 3.2
T_p	wave period between adjacent peaks
$\bar{T}_p$	mean wave period
T_v	visual estimate of wave period
T_z	zero crossing period
$\bar{T}_z$	mean zero crossing period
V_s	velocity of the ship
V_w	wind velocity
$W(f)$	energy spectrum in terms of cyclic frequency (subscript denotes measured parameter)
x	an arbitrary parameter
x_D	estimate of the extreme value for design purposes
$\{x_r\}$	discrete time series
X_i	extreme response
α	empirically derived coefficient for standard wave spectra
α_0	wave slope amplitude
β	empirically derived coefficient for standard wave spectra
$\Gamma(\lambda)$	the gamma function

γ	risk parameter for long term probabilities
Δt	discrete time interval, inverse of digitising frequency f
$\delta\phi$	directional spreading band
$\delta\omega$	circular frequency interval
ε	bandwidth parameter
ζ	wave elevation
ζ_0	wave amplitude
$\bar{\zeta}_0$	mean wave amplitude
ζ_m	most probable (modal) wave amplitude
λ	a shape parameter used with the Ochi 6-Parameter Spectrum
η	maximum$(X_1, X_2, ..., X_n)$
μ	the mean value of measured a parameter x
ξ	non-dimensional parameter x$/ \sigma$
π	pi, 3.14159
ρ	density of sea water
σ	the standard deviation of measured a parameter about it's mean
ϕ	spreading angle
ψ	relative heading angle between ship's heading and direction of waves
ω	circular wave frequency
$\bar{\omega}$	average circular frequency
ω_e	encounter frequency at which the ship encounters waves
$\omega_{e\,max}$	maximum encounter frequency for given conditions
ω_k	discrete circular wave frequency

REPORT NO. DSTO-TR-0183	AR NO. AR-009-303	REPORT SECURITY CLASSIFICATION UNCLASSIFIED

TITLE

Ship structural response analysis spectra and statistics

AUTHOR(S)
B. Phelps

CORPORATE AUTHOR
DSTO Aeronautical and Maritime Research Laboratory
PO Box 4331
Melbourne Victoria 3001

CLASSIFICATION/LIMITATION REVIEW DATE	CLASSIFICATION/RELEASE AUTHORITY Chief, Ship Structures and Materials Division

SECONDARY DISTRIBUTION

Approved for public release

ANNOUNCEMENT

Announcement of this report is unlimited

KEYWORDS

Wave analysis
Extreme loads
Long term probabilities

Spectral analysis
Structural response
Response amplitude operators

Loads (forces)
Fatigue

ABSTRACT

Wave loading of ships and maritime structures is a random process and the response of these structures is itself a random process. Much work has been undertaken to better understand the nature of waves and a number of existing techniques are described. Statistical and spectral analysis techniques may be used to quantify the wave loads and the corresponding structural responses. The relationship between the input load and response may then be determined through the use of response amplitude operators and the response of the structure to predicted lifetime extreme loads and fatigue loading then calculated. Standard wave spectra and directional spreading factors are used to enable the seaway to be described mathematically and several standard forms are discussed. The use of statistical methods enables the seaway to be described in terms of a limited number of parameters from which short term and long term probability distributions may be obtained. These distributions may then be used to enable extreme lifetime loads to be estimated.

Ship Structural Response Analysis: Spectra and Statistics

B.P. Phelps

(DSTO-TR-0183)

DISTRIBUTION LIST

Director, AMRL
Chief, Ship Structures and Materials Division
Dr J.C. Ritter
Mr B.P. Phelps   4 copies
Task Sponsor (DNA)   2 copies
Library, AMRL Maribyrnong
Library, AMRL Fishermens Bend

Chief Defence Scientist (for CDS, FASSP, ASSCM)   1 copy only
Head, Information Centre, Defence Intelligence Organisation
OIC Technical Reports Centre, Defence Central Library
Officer in Charge, Document Exchange Centre   8 copies
Air Force Scientific Adviser, Russell Offices
Senior Defence Scientific Adviser / Scientific Adviser - Policy and Command   1 copy
Senior Librarian, Main Library DSTOS
Librarian, DSD, Kingston ACT
Serials Section (M List), Deakin University Library, Deakin University, Geelong 3217
ABCA, Russell Offices, Canberra ACT 2600   4 copies
Librarian, Australian Defence Force Academy
Head of Staff, British Defence Research and Supply Staff (Australia)
NASA Senior Scientific Representative in Australia
INSPEC: Acquisitions Section Institution of Electrical Engineers
Head Librarian, Australian Nuclear Science and Technology Organisation
Senior Librarian, Hargrave Library, Monash University
Library - Exchange Desk, National Institute of Standards and Technology, US
Acquisition Unit (DSC-EO/GO), British Library, Boston Spa, Wetherby, Yorkshire LS23 7BQ, England
Library, Chemical Abstracts Reference Service
Engineering Societies Library, US
Documents Librarian, The Center for Research Libraries, US
NAPOC QWG Engineer NBCD c/- DENGRS-A, HQ Engineer Centre, Liverpool
    Military Area, NSW 2174 - data sheet of Army sponsored tasks only
Army Scientific Adviser, Russell Offices - data sheet only
Navy Scientific Adviser - data sheet only
Director General Force Development (Land) - data sheet only
ASSTASS PD, CP3-4-12, Campbell Park Offices, Canberra ACT - data sheet only
SO (Science), HQ 1 Division, Milpo, Enoggera, Qld 4057 - data sheet of Army sponsored tasks only
Librarian - AMRL Sydney - data sheet only
Counsellor, Defence Science, Embassy of Australia - data sheet only
Counsellor, Defence Science, Australian High Commission - data sheet only
Scientific Adviser to DSTC Malaysia, c/- Defence Adviser - data sheet only
Scientific Adviser to MRDC Thailand, c/- Defence Attache - data sheet only

Australian Maritime Engineering CRC:
 Melbourne Node, Monash University, Wellington Rd, Clayton Vic 3168
 (Attention: Prof J. Hinwood)
 Perth Node, Curtin University of Technology, GPO Box U1987, Perth WA 6001
 (Attention: Mr K. Klaka)
 Perth Node, Curtin University of Technology, GPO Box U1987, Perth WA 6001
 (Attention: Prof J. Penrose)
 BHP Research Melbourne Laboratories, Wellington Rd, Mulgrave Vic 3170
 (Attention: Mr Peter Mutton)